Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

24 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

577

A003

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

8824 – 9700

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

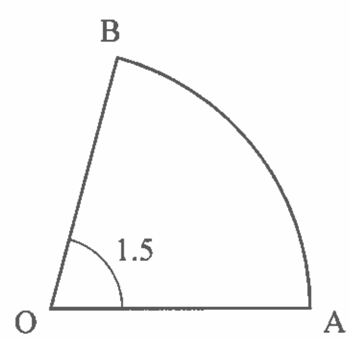
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $\mathrm{A\hat{O}B} = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $48\,\mathrm{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.45$ and $\mathrm{P}(A \cup B) = 0.85$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Find $\mathrm{P}(A' \mid B')$. [3]

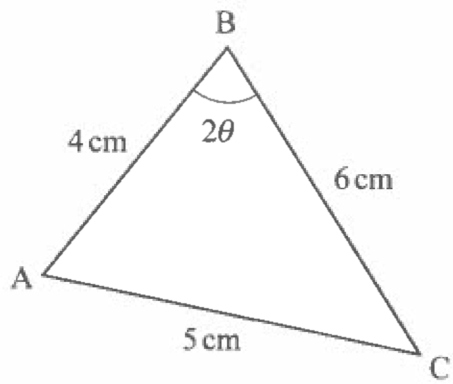
3. [Maximum mark: 4]

Prove that $(3n+2)^2 - (3n-2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

4. [Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 4\,\text{cm}$, $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $A\hat{B}C = 2\theta$.

diagram not to scale



Find the exact value of $\cos\theta$, giving your answer in the form $\dfrac{p\sqrt{2}}{q}$, where $p, q \in \mathbb{Z}^+$.

5. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

6. [Maximum mark: 5]

(a) Solve $2x^2 - 15x + 18 < 0$. [3]

(b) The function f is defined by $f(x) = \sqrt{2x^2 - 15x + 18}$, where $x \in \mathbb{R}$, $x \leq k$.

Find the greatest value of k for which f^{-1} exists, justifying your answer. [2]

7. [Maximum mark: 7]

Consider the function $f(x) = \sec\left(x - \frac{\pi}{4}\right)$, for $0 \le x \le \frac{\pi}{2}$.

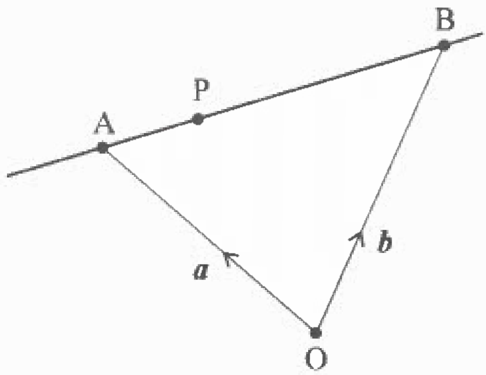
(a) Determine the range of f. [3]

The region bounded by the graph of $y = f(x)$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{2}$ is rotated 2π radians about the x-axis.

(b) Find the volume of revolution generated. [4]

8. [Maximum mark: 8]

The following diagram shows two points A and B such that $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$.



The point P lies on (AB) so that $\overrightarrow{AP} = \lambda\,\overrightarrow{AB}$ where $0 < \lambda < 1$.

(a) Show that $\overrightarrow{OP} = (1-\lambda)\boldsymbol{a} + \lambda\boldsymbol{b}$. [1]

It is given that $|\boldsymbol{a}| = 1$, $|\boldsymbol{b}| = 2$ and $\boldsymbol{a}\cdot\boldsymbol{b} = \frac{1}{4}$.

(b) In the case that $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{AB}$, find the value of λ. [7]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

9. [Maximum mark: 9]

(a) Prove that $\tan\left(\theta - \frac{\pi}{4}\right) \equiv \frac{\sin 2\theta - 1}{\cos 2\theta}$, where $\theta \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$. [6]

(b) Hence, or otherwise, solve $\frac{\sin x - 1}{\cos x} = \sqrt{3}$ for $0 \leq x \leq 2\pi$. [3]

Do **not** write solutions on this page.

Section B

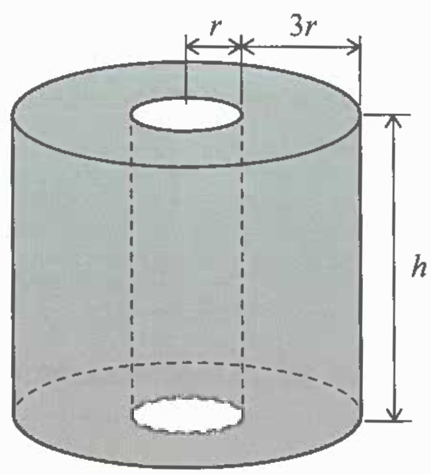
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi\,\text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

A curve is given by the equation $y = \frac{e^{2x} - 1}{e^{2x} + 1}$, $x \in \mathbb{R}$.

(a) By applying l'Hôpital's rule or otherwise, show that $\lim_{x \to \infty} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right) = 1$. [2]

(b) (i) Show that $\frac{dy}{dx} = \frac{4e^{2x}}{\left(e^{2x} + 1\right)^2}$.

(ii) Hence, show that $1 - y^2 = \frac{dy}{dx}$. [6]

(c) (i) By using implicit differentiation and the result in part (b)(ii), show that $\frac{d^2y}{dx^2} = 2y^3 - 2y$.

(ii) Hence, find an expression for $\frac{d^3y}{dx^3}$ in terms of y. [5]

(d) By using your results from parts (b) and (c), find the Maclaurin series for $\frac{e^{2x} - 1}{e^{2x} + 1}$ up to and including the term in x^3. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 20]

Consider the equation $z^4 = 16\mathrm{i}$, where $z \in \mathbb{C}$.

The equation has four roots z_1, z_2, z_3, z_4, where $z_i = r(\cos\theta_i + \mathrm{i}\sin\theta_i)$, $r > 0$ and $0 \le \theta_1 < \theta_2 < \theta_3 < \theta_4 < 2\pi$.

(a) Find z_1, z_2, z_3 and z_4. [6]

The roots z_1, z_2, z_3 and z_4 form a geometric sequence.

(b) Find the common ratio of the sequence, expressing your answer in Cartesian form. [3]

The roots z_1, z_2, z_3 and z_4 are represented by the points A, B, C and D respectively on an Argand diagram.

(c) Plot the points A, B, C and D on an Argand diagram. [3]

The equation $v^4 = a + b\mathrm{i}$, where $v \in \mathbb{C}$ and $a, b \in \mathbb{R}$ has roots z_1^*, z_2^*, z_3^* and z_4^*.

(d) Determine the value of a and the value of b. [3]

The midpoint of $[\mathrm{AB}]$ is A', the midpoint of $[\mathrm{BC}]$ is B', the midpoint of $[\mathrm{CD}]$ is C' and the midpoint of $[\mathrm{DA}]$ is D'.

Consider the equation $w^p = 2^q$, where $w \in \mathbb{C}$ and $p, q \in \mathbb{Z}^+$.

Four of the roots of $w^p = 2^q$ are represented by the points A', B', C' and D'.

(e) Find the least possible value of p and the corresponding value of q. [5]